PROJECTION OF HOMEBROKER PENETRATION PROJECTION WITHIN THE BRAZILIAN CAPITAL MARKET VIA LINEAR, FISHER-PRY AND GOMPERTZ MODELS

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ABSTRACT

In this paper, the penetration projection of the Home Broker tool in the servicing of mass market customers of the Brazilian stock market, up to 2012, is calculated. Estimates were prepared using three different models: Linear, Fisher-Pry and Gompertz. The three projections were compared to each other both by the coefficients of determination of the estimates and by the resulting absolute and squared deviations. Since the Fisher-Pry model adds the characteristic of modelling technological dynamics as a function of the market percentage conquered or yet to conquer – as opposed to the Gompertz model, whose technological penetration is exclusively influenced by the market to be the conquered and the Linear model might be perceived as a simplistic case amongst the three alternatives – results indicated, as expected given the specificities of the case studied case, that the projection using the Fisher-Pry model best adheres to data, indicating that, by the end of 2012, approximately 84% of individuals that negotiate at the stock market shall execute their orders through Home Broker. Irrespective of the toughness of statistical results, it´s worth emphasizing that projections effected might prove to be underestimated, given that the models employed do not comprise the possibility of other innovations on the market, such as the recent development of the *Direct Market Access* (DMA) software.

Key-words: Technology Substitution. Growth Curves. Fisher-Pry. Gompertz. Home Broker.

PROJEÇÃO DA PENETRAÇÃO DO HOME BROKER NO MERCADO DE CAPITAIS BRASILEIRO ATRAVÉS DOS MODELOS LINEAR, FISHER-PRY E GOMPERTZ

RESUMO

Neste trabalho, calcula-se a projeção da penetração da ferramenta *Home Broker* no atendimento de clientes pessoas físicas no mercado acionário brasileiro até 2012. Foram realizadas estimações com três diferentes modelos: *Linear, Fisher-Pry* e *Gompertz*. As três projeções foram comparadas entre si tanto pelos coeficientes de determinação das estimações, quanto pelos desvios absolutos e quadráticos produzidos. Como o modelo *Fisher-Pry* carrega a característica de modelar a dinâmica tecnológica como função do percentual de mercado já conquistado e a ser conquistado - diferentemente do modelo de *Gompertz*, cuja penetração tecnológica é influenciada exclusivamente pelo mercado a ser conquistado, e o modelo *Linear* pode ser visto como um caso simplista entre as três alternativas - os resultados indicaram, como esperado dadas as especificidades do caso estudado, que a projeção pelo modelo *Fisher-Pry* é a mais aderente aos dados, indicando que, no final de 2012, cerca de 84% das pessoas físicas que negociam em bolsa devem executar suas ordens através do *Home Broker*. Apesar da robustez dos resultados estatísticos, ressalta-se que as projeções realizadas podem estar subestimadas, uma vez que os modelos empregados não contemplam a possibilidade de outras inovações no mercado, como o recente desenvolvimento do *Direct Market Access* (DMA).

Palavras-chave: Substituição Tecnológica. Curvas de Crescimento. *Fisher-Pry*. *Gompertz* . *Home Broker*.

1 INTRODUCTION

Prospective analyses are important tools in corporate strategic planning. Since the object of study of such research impact market dynamics, giving rise to new business modalities, knowing in which direction or at what pace a chosen technology will evolve might be vital to the maintaining of a company´s competitive advantages.

According to Coates et al. (2001, p.8), "companies now realize that technological decisions are business decisions much the same as are financial, marketing or other strategic decisions". In view of the recognized influence of technologies in the development of a country, prospective analysis might support decisions concerning a country´s growth policies (Johnson & Marcovitch, 1994).

In general, prospective methods might be divided into three large groups: structural, correlative and direct (Porter, 1991). Amongst direct methods are estimates known as S growth curves, which include the Fisher-Pry and Gompertz models. Whilst the first is considered a typical growth model, the second is known as a mortality model.

Despite the clear distinction between their applications, usually both produce results which are, statistically, very close (Trappey & Wu, 2008). Therefore, it would be valid to compare parameters obtained in the estimates of both models, in addition to the Linear model, as of a case whereby, typically, the Fisher-Pry growth model is more adequate to describe the penetration of technology in comparison to alternative models.

The focal technology of this study is the Home Broker (utilized in meeting the needs of individual investors of the Brazilian capital market) that experienced significant innovations during the course of its history, bringing about greater agility to the manner how brokers contact their clients and in the consequent expansion of business.

Since the emergence of the *Computer Assisted Trading System* (CATS) - an electronic negotiation system that operated in conjunction with the hands free device at some markets from 1990 to 1997, having later been substituted by "MegaBolsa" – the volume of negotiations increased at high rates. Irrespective of

the financial and commercial freedom that took place during the period having influenced this growth in the national capital market, one cannot refrain from acknowledging that technological innovation also played a relevant role. To illustrate this statement, evolution of the percent of electronic business and number of monthly transactions at "Bovespa" are presented on Graph 1.
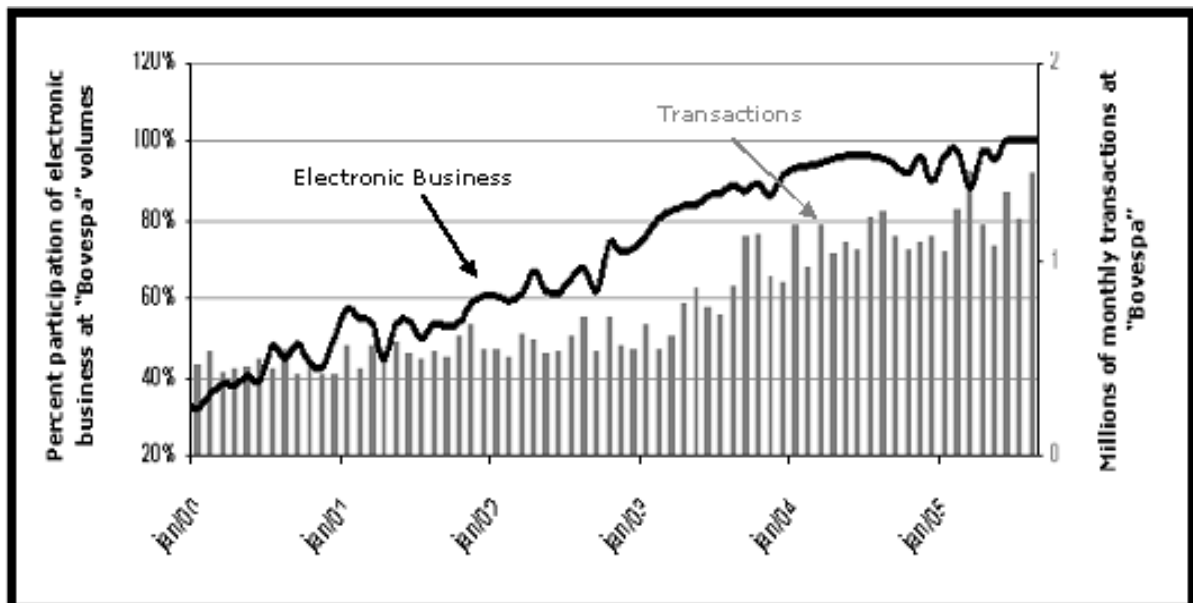
Graph 1: Percent participation in electronic business and evolution of the number of monthly transactions at "Bovespa"

Source: www.bovespa.com.br

At the end of the XIX century, contact between brokers and their clients was predominantly face-to-face. On the other hand, the XX century housed the consolidation of the telephone and of the internet. Lately, Home Broker became the prime means of negotiation for individual investors in the Brazilian stock market. In Graph 2, one visualizes the evolution of the number of individuals holding active business at "Bovespa", which surpassed slightly more than 5 thousand in the beginning of the year 2000, to almost 200 thousand in 2009, representing an approximately 4.000% growth rate during the period. In addition to this, one notes a vigorous raise in the participation of business conducted over the Home Broker platform, which started in April 1999*.* Given the above, it seems reasonable to suppose that growth in the participation of individuals on the market is associated with the availability of the tool for negotiations over the internet.
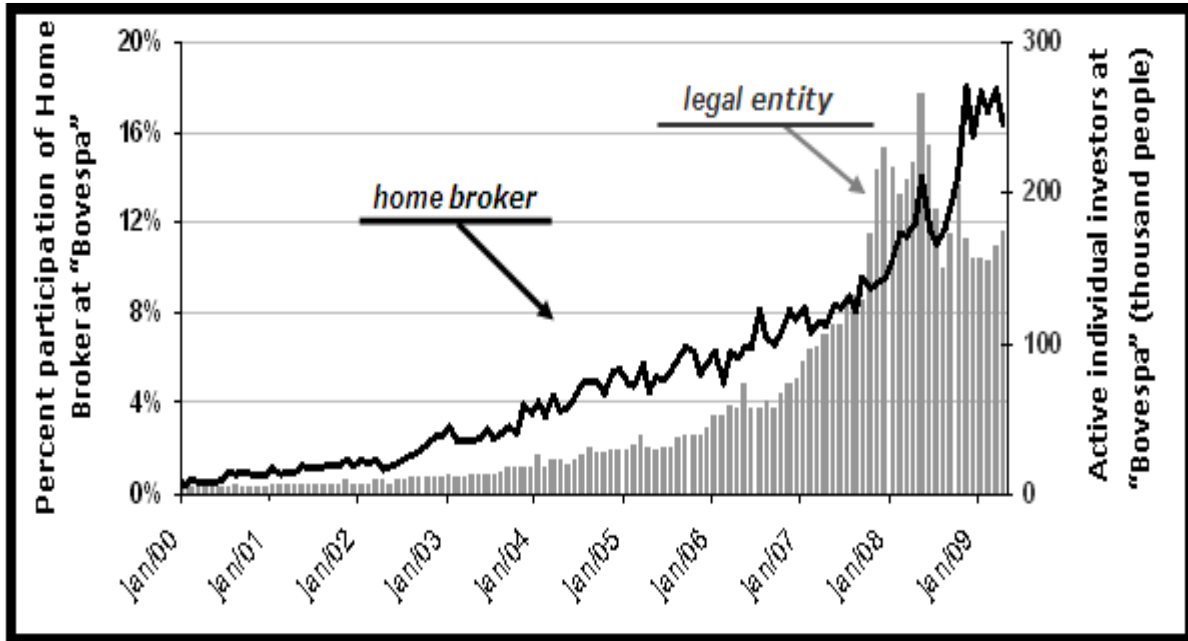
Graph 2: Participation of individual investors and of the Home Broker
technology in the volume at "Bovespa"

Source: www.bovespa.com.br

Broking companies would like to know of the future evolution of the participation of the new technology as a means of business with their individual clients. Under this context, given the reduced empirical exploration of this innovation in Brazil, the current study poses to contribute performing projections concerning the Home Broker´s technological penetration to service individuals at Brazilian brokers up to the end of 2012.

To meet the proposed objective, this research paper is divided into 5 sections, including this introduction. In the second section, a short bibliographical revision of innovations that occurred in the capital market is presented. The third section is dedicated to the exposure of models utilised in estimates, *i.e.*, *Fisher-Pry*, Gompertz and Linear, in addition to the description of data utilised. Results are exposed in the fourth section. In the fifth and last section, final considerations are presented.

2 CAPITAL MARKET TECHNOLOGICAL EVOLUTION

As with other technologies and businesses, the emergence of virtual brokers started off in the US. In 1994, Aufhauser Co. started their online services, rapidly followed by other technology, communications and financial

services firms (E.G, CompuServe and AOL). The rapid adoption of the new tool changed the competitive dynamics of the market, which previously was segmented into full service brokers and discount brokers. Whilst the first offered different investment services such as analysis of companies and personalized servicing, discount brokers focused on the provisioning of low cost transactions.

In 1990, full service brokers held 84% of the North American market. The resistance of some of these companies in offering the new business technology to their clients, moreover given the price war that occurred in the online segment, reduced their participation on the market to merely 5% in 1999 (Gaudillat & Quélin, 2006).

Literature indicates some causes for the rapid expansion of the online stocks market in the US. Dasgupta (1998) attributes the success of these businesses to the abundance of the North American economy which increased at high rates during the three year period of 1994-1997. For Gaudillat and Quélin (2006), growth was mostly due to the high marketing expenses experienced by new entrants.

The importance of online transactions is not only in terms of business done via face-to-face activities. In 1998, more people purchased online stocks than books or CDs, using the same distribution channel. For Dasgupta (1998), the growth of online business attracted new investors to the capital market, explaining the significant increase in the participation of individual investors which, in the year 2000, already captured 27% of total businesses in the US. Li, Lee and Cude (2002) evaluated some business clients online, concluding that the adoption of new business technologies via the internet is most likely to occur amongst younger people, less adverse to risk and customers of discount brokers.

In Brazil, significant technological evolutions also took place during the course of "BM&F Bovespa´s" development. For instance, the registry of transactions with stocks changed from the manual method to perforated cards, thereafter to optical reading cards, until being substituted by 100% electronic transactions by the time hands free auctions ended in 1995.

In a similar manner, broker contact with customers went from the stage orders were written or verbally transmitted, to those processed over the phone and, more recently, with the emergence of the Home Broker - a tool provided by "BM&FBovespa" that enables business involving stock to be done over the internet – likewise being effected, within a virtual environment. Therefore, it's probable that by the mid 90´s, transaction costs ought to have been higher than

at present, for both the companies that had to hire in person brokers and customers, also had to wait several precious minutes to be answered over a call, which often corresponded to loss in terms of an opportunity of closing a lucrative transaction.

Souza and Cova (2009) evaluated and identified, through qualitative analysis, influences of changes promoted by the implementation of the Home Broker on agents. In addition to the reduction of operational costs, authors indicate that the Home Broker enabled negotiations beyond conventional working hours, further extending the client base, allowing contributors to incorporate smaller investors that previously did not trade on the market.

Technological evolutions alter the way brokers' position strategically within the domestic market, where development of companies focused on servicing via the internet, may be observed. In October 2009, the volume of business closed using the Home Broker surmounted 19% of "Bovespa´s" total, with over 68 brokers offering online services.

There are other advances that, despite not as yet constituting a technological mainstream, are observed at the Brazilian stock market. In 2009, "BM&F Bovespa" authorized the adoption of a technology known as *Direct Market Access* (DMA) model 3, which enables clients to electronically trade without having their orders processed by brokers despite these still being legally responsible for operations.

In the same year, some *algotraders* - investors with businesses programmed by algorithms - were able to physically install their suppliers at "BM&F Bovespa´s" environment, further increasing the speed of order processing. It is reasonable to affirm that the emergence of the virtual environment and other technological innovations must elevate the level of uncertainty concerning the future of the market. In this sense, prospective analysis gain relevance as a tool to support strategic decisions at companies.

3 METHODOLOGY

3.1 EXTRAPOLATION MODELS: LINEAR, FISHER-PRY AND THAT OF GOMPERTZ .

Technological prospection methods may be classified into structural, of correlation and direct (Porter, 1991). In the first group, a technological projection is performed as of factors related to the referred advance, using

statistical models such as simulation and regression analysis. In the second group is the scenario, analogies and cross-impact methods. The third group contemplates the Dephi technique, time series estimates and trend extrapolations.

New prospection methods arise along time whilst some traditional ones are perfected. For instance, scenario building has been incremented with the implementation of computing methods that support its construction (E.G, Gausemeier et al., 1998, *apud* Coates et al., 2001). Disseminated as of the communist opening at the end of 1980, the theory of the inventive solution of problems, known under the acronym TRIZ, has been conquering popularity. The Delphi also gained agility and its once high preparation costs are falling with the use of computerized methods for the treating of data and internet as a means of issuing questionnaires. In addition to these, there are monitoring methods that rely on bibliometric research and roadmaps, which are both worthy of mention.

The notion of an adequate prospective method is relative. Thus, the use of one or another method, whether old or new, generated as of the re-emergence of prospective analysis in the 90´s, depends on the technical knowledge of researchers, on the interests which are intrinsic to the conducted study and specificities of the evaluated technology. If, on one hand, for the projection of the distant future of a nation, great effort and resources would be required, whereby the joint use of an assortment of methods is suggested; research concerning the penetration of a new technology on the market under a shorter term horizon might be conducted by less experienced researchers, with fewer resources and in less time. To this effect, projections per trend extrapolations are particularly emphasized given the ease and speed of use using computer resources and as of a reduced set of observations.

The most simple trend extrapolation model, classified by some authors as a structural model (Coates et al., 2001) is the Linear model which can be represented by the equation:

$$(1) \quad P = b_{LP} + z_{LP}.t \, ,$$

whereby P is the market share conquered by the new technology,, t is a time variable, and b_{LP} and z_{LP} are parameters to be estimated, for instance, by ordinary least squares.

Trend extrapolation may also be performed by growth curves known as family S curves, which represent the growth periods, inflection and product or new technology saturation. In these models, the penetration of a given technology or new product grows at increasing rates until its point of inflection, when penetration starts growing at decreasing rates, converging to its maximum point, saturation point or maximum limit that the market can cope with.

Two widespread S curve models were utilized in this paper: Fisher-Pry and Gompertz. The Fisher-Pry model provides a feature that models the growth dynamics of the new technology as a function of the market share to conquer and also that of the already conquered market. The Gompertz model, on the other hand, has penetration exclusively influenced by the share of the market to be conquered.

The existence of "word of mouth" effects concerning the use of the Home Broker ensures the increase in the number of clients induces an additionally greater elevation in its penetration. Thus, given the intuitions of both models, it seems reasonable to assume that Fisher-Pry is more appropriate to the case under study. Mathematically, the Fisher-Pry model establishes the growth of user percentages, i.e*. dP/dt*, according to:

$$(2) \quad \frac{dP}{dt} = z.P\left(1 - \frac{P}{K}\right)$$

whereby P is the percent of individual investors who utilize the Home Broker, K is the maximum limit of users supported by the new technology and z is named as a proportionality constant. Solving the above equation (see details in attachment I), one comes to:

$$(3) \quad P = \frac{K}{1 + e^{(-b-zt)}},$$

whereby b and z are parameters to be estimated. The first, b, is a constant that arose in the equation solution process (2) and the fraction $-b/z$ represents the inflexion point of the growth rate, when the number of users of the technology starts to grow at smaller rates. To project the Home Broker percent on the market, the assumption is that in the future, all individual investors shall utilize the technology. Thus, it is established that K is equivalent to 1.

Known as the mortality model, the Gompertz method is meant to be used in cases where the adoption of the new product depends on the exhaustion of the old one. For instance, it is easier to admit that people change their television sets for new and more modern devices when the old one start presenting defects that represent a high maintenance cost, inducing the technological exchange. In this sense, the Gompertz model would be appropriate if the negotiation via Home Broker should refrain the client from negotiating over the telephone. This doesn´t seem to be the case for a same client may have orders transmitted over both the telephone and the internet.

In the Gompertz model, the assumption is that the growth in market share rate, *dP/dt*, occurs according to the following function:

$$(4) \ \frac{dP}{dt} = z.P.\ln\left(\frac{K}{P}\right)$$

whereby, as in the Fisher-Pry model, *K* is the maximum limit of users supported by the new technology and *z* is a proportionality constant. Solving the above equation (see details in attachment II), one comes to:

$$(5) \ P = K.e^{-e^{-z(t-b)}}$$

whereby *z* and *b* are parameters to estimate. As in the Fisher-Pry model, the assumption is that the percentage of individual investors that will utilize the Home Broker in the future is 100%. Thus, it is established that *K* equals 1.

The logistic equation estimates (3) and (5) were calculated utilizing the interactive method which, as of tentative initial values, uses the minimum ordinary square method until there are no substantial alterations in the parameters (Wooldridge, 2000). Econometric procedures were performed using STATA 9.1. software.

3.2 MODEL EVALUATION

To comparatively evaluate the three estimates performed (equations 1, 3 and 5), absolute deviance statistics (DA) and square deviances (DQ) for each one were calculated as of equations (6) and (7) below:

$$(6)\ DA_{x,t} = |P_t - \hat{P}_{X,t}|$$

$$(7)\ DQ_{x,t} = (P_t - \hat{P}_{X,t})^2$$

whereby P_t represents the effective percent of the technology in time *t* and $\hat{P}_{X,t}$ is the estimate calculated by the *X* method, where *X* equals 1 (Linear), 3 (*Fisher-Pry*) or 5 (Gompertz). Note that the difference $(P_t - \hat{P}_{X,t})$ is the residue of the estimate by *X*. Thus, the smaller the *DA* and *DQ* statistics, the better are the estimates produced by the method.

To test the hypothesis that the Fisher-Pry (3) model is more adequate to the case studied than the Linear (1) and the Gompertz (5) model, t tests were conducted comparing pairs of average deviances. Thus, the following null hypotheses were tested:

$$H0_1: \overline{DA_1} \leq \overline{DA_3}$$

$$H0_2: \overline{DA_5} \leq \overline{DA_3}$$

$$H0_3: \overline{DQ_1} \leq \overline{DQ_3}$$

$$H0_4: \overline{DQ_5} \leq \overline{DQ_3}$$

Rejecting null hypothesis will mean that the Fisher-Pry model is not inferior to the alternative models.

3.3 DATA UTILIZED

Data utilized was extracted from "BM&F Bovespa´s" website and is composed of monthly information for the period between April 1999 and April 2009. "Bovespa´s" total percent volume was obtained (*i.e.*, spot, forward, future and options markets) as negotiated over Home Broker (PERCVOLHB); in addition to the percent participations of volumes negotiated by individual investors, likewise over the total market (PERCVOLPF).

Table 1: Summary of series utilized

YEAR	PERCVOLPF*	PERCVOLHB*	PFHOME**	PFTELE**
2.000	19,13%	0,71%	3,73%	96,27%
2.001	20,53%	1,14%	5,57%	94,43%
2.002	20,79%	1,71%	8,20%	91,80%
2.003	24,17%	2,79%	11,54%	88,46%
2.004	27,53%	4,46%	16,20%	83,80%
2.005	25,39%	5,47%	21,55%	78,45%
2.006	24,67%	6,77%	27,42%	72,58%
2.007	23,16%	8,44%	36,47%	63,53%
2.008	26,84%	12,87%	47,94%	52,06%
2.009#	32,81%	17,39%	53,00%	47,00%
Average	24,5%	6,2%	23,2%	76,8%
Standard Deviation	4,0%	5,4%	17,6%	17,6%

Key: (#) up to April 2009; PERCVOLPF: percent of "Bovespa´s" total volume relative to individual investors; PERCVOLHB: percent of "Bovespa´s" total volume pertaining to Home Broker; PFTELE: percent of individual investors served over the telephone; and PFHOME: percent of the volume of individual investors served via Home Broker.

Sources: (*) www.bovespa.com.br - São Paulo Stock Exchange Market website; (**) calculated by the author

Given the assumption that all business effected via the Home Broker is performed by individual investors and that these negotiate exclusively via this tool or the telephone, one might infer the percents of the new technology´s penetration on these clients, creating the variable PFHOME, built as of the division of the PERCVOLHB by PERCVOL. The PFTELE variable, percent of business of individual investors done over the telephone, was obtained subtracting PFHOME from 100%. Table 1 presents the annual averages of the four variables. In 2009, for instance, 53% (rounding up, 17,4% ÷ 32,8%) of the volume negotiated by individual investors was so done over the Home Broker platform and 47% (100% - 53%) over the telephone.

4 PRESENTATION AND ANALYSIS OF RESULTS

Equations (1), (3) and (5) were estimated following the previously described methods, using PFHOME as an endogenous variable. The three models prove to be very adhering to data, however, generated significant differences in projections for 2012. Whilst the Linear projection indicated that, in December 2012, the Home Broker will be the chosen servicing method for 64,58% of individual "Bovespa" investors, the projection via Fisher-Pry suggests an 83,75% percent and Gompertz´s projection 72,82%.

The complete estimate results can be visualized in Table 2. All coefficients of the three equations are statistically different from zero with significance levels that are inferior to 1%. Furthermore, all coefficients of determination (R^2) prove to be higher than 90%.

Table 2: Estimate results (Linear**,** Fisher-Pry ***and*** Gompertz **)**

DEPENDING VARIABLE: HB	LINEAR		FISHER-PRY		GOMPERTZ	
	COEF.	ESTAT T	COEF.	ESTAT T	COEF.	ESTAT T
k	---	---	1	---	1	---
b	0,00428	30,34	0,03252	76,83	0,01692	53,02
z	-0,1816	-15,06	142,59	342,58	125,12	238,09
R^2		0,9176		0,9960		0,9916
F Statistics		920		14862		7053
Number of observations		121		121		121

Source: author estimates

It´s worth noting that the determination coefficients of the three estimates are comparable given that the endogenous variable is the same in the three models and that these were estimated under the same number of observations. Considering that the Linear model presents the lowest and that Fisher-Pry provides the highest determination coefficient, one might state that they represent the worst and the best projection, respectively.
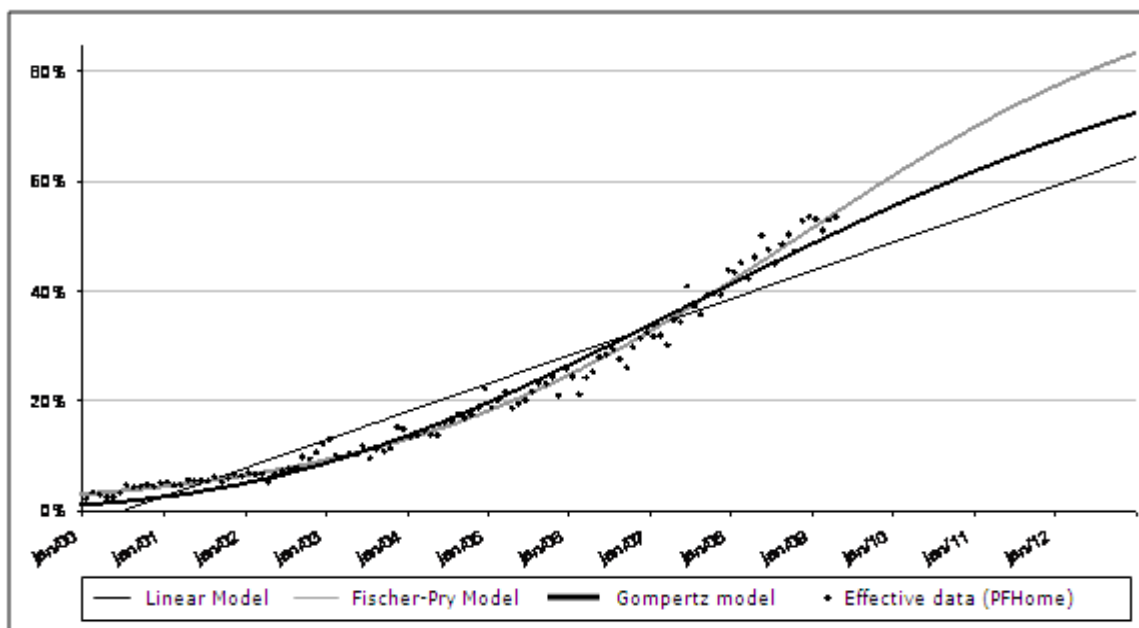
Graph 3: Linear, Fisher-Pry and Gompertz model projections
Source: author calculations

From Graph 3 one might visualize the values foreseen by the Linear, Fisher-Pry and Gompertz models. The visual analysis is in line with the comparison of the three models performed by the determination coefficient. Apparently, the Fisher-Pry model generated the best projection and the Linear one, the worst. Subsequently, the statistical averages of the absolute and square deviances (*DA* e *DQ*) were tested. Summarized results are presented in Table 3. From the tests applied, all four null hypotheses are rejected at a significance level of less than 1%, corroborating the best fit of the Fisher-Pry model to Home Broker´s penetration in the local market. Absolute deviances indicate similar results to those encountered, so far. Square deviances, in turn, indicate the Gompertz model as being the worst option.

Table 3: Average absolute and square deviance tests

NULL HYPOTHESIS	AVERAGE ABSOLUTE $(\overline{DA_X})$ AND SQUARE DEVIANCES $(\overline{DQ_X})$		AVERAGE DIFFERENCES STATISTICAL T (*)
$H0_1$:	$\overline{DA_1}$	$\overline{DA_3}$	13,54
	0,0387	0,0117	
$H0_2$:	$\overline{DA_5}$	$\overline{DA_3}$	6,73
	0,0184	0,0117	
$H0_3$:	$\overline{DQ_1}$	$\overline{DQ_3}$	9,70
	0,0020	0,0002	
$H0_4$:	$\overline{DQ_5}$	$\overline{DQ_3}$	6,04
	0,0052	0,0002	

(*) All tests were performed with 121 observations or 120 degrees of freedom. The four null hypothesis tested are rejected at a level of significance which is less than 1%.

Source: author´s estimates

5 FINAL CONSIDERATIONS

Here, a brief evolution of stock transactions using Home Broker software, emphasizing the relevance of the emergence of online business in the expansion process concerning capital market individual investors, was presented. To perform the short term projection of the Home Broker technology´s penetration amongst individual clients, the trend extrapolation method occupies a distinctive position given that it is easily implemented by means of computers.

Amongst major extrapolation models found in literature there are those of Fisher-Pry and Gompertz. Whilst the first is appropriate in cases where the new technology offers clear advantages over the old technology, the second is associated with mortality studies, whereby penetration dynamics only depends on the market share to be conquered.

For the case studied in this paper, the assumption was that the Fisher-Pry model is the best intuitive match. Furthermore, one might state that the Linear model is a simplistic alternative as compared to the two models. As expected, given the dynamics of the technology evaluated, *Fisher Pry´s* logistic equation better explains the penetration of the Home Broker as a means of providing services to individual investors at the local market as opposed to Linear and Gompertz projections. Such a result was obtained by both visual examination and by comparing the determination coefficients of the models, and also using statistical average absolute and square deviances tests.

To project the trend of Home Broker´s participation as a means of providing services at broker companies trading Brazilian shares using *Fisher-Pry*, findings indicated that the referred channel will grow, by the end of 2012, on average, at 1% a month. This means that, in December 2012, Home Broker shall dominate approximately 84% of the individual investor market. Considering a daily volume of R\$ 5 billion at "Bovespa" and, should individual investors account for 35% of the total volume in 2012, there would be an increment of R\$430 million in daily business closed over the Home Broker platform in comparison to May 2009 levels.

It´s worth noting that, in this study, numerous factors which influence the expansion of the Home Broker were not taken into account – such as greater access of people to microcomputers and broad band connections and the fall in the price of services offered, which suggests that the projection prepared may prove to be an underestimate. Furthermore, the dissemination of new Technologies such as the *Direct Market Access* (DMA) might increment the Home Broker growth rate forecast in this study and also possibly modify the competitive dynamics at broker companies.

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ATTACHMENT I - FISHER-PRY MODEL

Consider *P* as the number of users of a given technology and *t* an independent time variable. The equation below (I) , called a *Logistic*, determines the growth rate of the number of users, *i.e.*, *dP/dt*:

$$(\text{I}) \quad \frac{dP}{dt} = z.P\left(1 - \frac{P}{K}\right)$$

whereby *K* is the maximum limit of users the technology handles and *z* the proportionality Constant. Should the number of users be small in relation to the ability to support them, *P/K* ➜ 0, then

$$\frac{dP}{dt} \approx z.P \, .$$

However, should the number of users come close to the support capabilty, *P/K* ➜ 1, then

$$\frac{dP}{dt} \to 0$$

Therefore, a curve with high growth rates at the start of the diffusion is formed, which tails off as it comes close to the limit the market is able to cope with. To solve the equation (I), one calculates

$$\int \frac{1}{P(1 - P/K)}dP = \int z.dt \, .$$

As

$$\frac{1}{P(1 - P/K)} = \frac{1}{P} + \frac{1}{K - P},$$

yielding

$$\int \left(\frac{1}{P} + \frac{1}{K - P}\right)dP = \int z.dt \, .$$

Solving the integration, then

$$P = \frac{K}{1 + Ae^{-zt}} \, , \quad \text{whereby} \quad A = \frac{K - P_0}{P_0} \, ,$$

or, alternatively,

$$P = \frac{K}{1 + e^{(-b-zt)}} \, ,$$

where *b* is a constant that arose during the integration process.

ATTACHMENT II - GOMPERTZ MODEL

Consider *P* as the number of users of a given technology and *t* an independant time variable. The equation below (II) establishes a growth rate for the number of users, *i.e.*, *dP/dt*:

$$(\text{II}) \quad \frac{dP}{dt} = z.P.\ln\left(\frac{K}{P}\right)$$

whereby once more, *K* is the maximum limit of users supported by the new technology and *z* is called a proportionality Constant. To solve equation (II), one sets

$$\int \frac{1}{P.\ln(K/P)} dP = \int z.dt \, .$$

Solving the integration above, yields

$$\ln[\ln(K/P)]\Big|_o^t = zt + f \, ,$$

Simplifying the expression, one comes to

$$P = K.e^{-e^{-z(t-b)}} \, ,$$

where b is a constant that arises during the integration process.